Computer Science IV Midterm Exam
September 2017
Indian Statistical Institute

All question references to Master's Theorem, use this

T (n) = a T (n/b) + f(n), where a, b and k be integers such that $a \geq 1$, $b \geq 2$ and $k \geq 0$, where f(n) is an asymptotically positive function

Part I - total 25 marks

1. Answer any 5 of the following - 2 marks each

a) The running time for typical graph algorithms using adjacency lists is …….. and adjacency matrix is ……
b) According to Master's theorem, Tn = O (n^k) if ……..
c) An algorithm where the immediate best solution is chosen at every step is called a ……. algorithm
d) Djikstra's algorithm finds the …….. in a graph
e) Huffman's code is an algorithm used for …….
f) Radix Sort is primarily used to sort …….

2. Answer true or false for any 5 of the following - 2 marks each

a) Huffman code finds the minimum spanning tree
b) The worst case time for Quick Sort is O (n)
c) Djikstra's algorithm is used to find the shortest path between any two vertices in a graph.
d) Fast Fourier Transform algorithm has important applications in digital signal processing.
e) Greedy coin changing algorithm is optimal for denominations 1, 5, 10 when the amount to be changed is 18
f) Partial solutions in Kruskal's algorithm always produce connected trees

3. Find a theta notation in terms of n for the number of times the statement x = x + 1 is executed in the segment - 5 marks

```
j = n
 while ( j ≥ 1)
 {
          for (i = 1 to j)
               x = x+1
          j = j / 2
 }
```

Part II Recurrence relations - total 25 marks

Definition: A recurrence relation for the sequence $a_0, a_1, \ldots a_n$, is an equation that relates a_n, to its predecessors $a_0, a_1, \ldots a_{n-1}$ with initial conditions explicitly specified for a finite number of terms of the sequence $a_0, a_1, \ldots$

1. **Obtain the recurrence relation for C_n - 5 marks**

   Let C_n denote the number of times the statement x = x +1 executes in the algorithm

```
func (n)
{
        if (n == 1) return
        for (i = 1 to n) x = x+1
        func (n/2)
}
```

   Obtain the recurrence relation for C_n in this case.

2. **Solve the recurrence relation - 10 marks**
   obtained for C_n in 2a above when n is a power of 2
   with initial condition $C_1 = 0$

3. **Solve the recurrence relation for Cn - 10 marks**
   $C_n = C_{n-1} + n$, $n \geq 1$, with initial condition $C_0 = 0$.

Part III Algorithms - 25 marks

1. **Solve T (n) = a T (n/b) + f(n) and f (n) in the following cases - 5 marks**
   1. a = 1, b = 2, k = 1, f(n) = n
   2. a = 2, b = 2, k = 1, f(n) = n
   3. a = 12, b = 3, k = 2, f(n) = n^2

2. **Prove that the Preorder algorithm for a binary tree with n nodes runs in theta (n) - 5 marks**

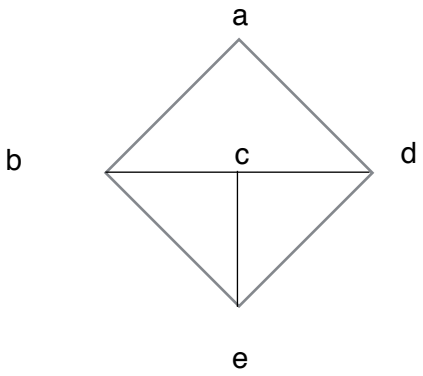
3. **Solve the following - 15 marks**

Given the coding sequence 000101100100111000101010010000101 given the following characters and their respective frequencies.
a – 25, s – 19, y – 6, t – 8, # – 2, o – 22, e – 34,

1. Build the Huffman tree
2. Calculate total bits for each character
3. Decipher the code

Part IV - Graphs - 25 marks

Consider the connected weighted graph below



Weights for the edges are
a, b = 3; a, d = 3; b,c = 1; b, e = 4; c, d = 2; c, e = 1; d, e = 3

1. **Trace Prim's algorithm for the above graph - 10 marks**
2. **Use Djikstra's algorithm - 15 marks**
   a. To trace a single source shortest path from vertex a to vertex e - 10 marks
   b. Write the pseudocode for a single source shortest path. State your assumptions - 5 marks